Rational Functions and One-Sided Limits (1.9-1.11) HW

Let $f(x)$ be a rational function that is of the form $f(x) = \frac{n(x)}{d(x)}$. Match each of the four numbered descriptions below with the four lettered descriptions to the right. Use each description exactly once. There are no repeats.

1. $n(x)$ has a zero at $x = 2$ and does not have a zero at $x = 5$. $d(x)$ has a zero at both $x = 2$ and $x = 5$. The multiplicity of the zeros at $x = 2$ is equal for $n(x)$ and $d(x)$

2. $n(x)$ has a zero at $x = 5$ and does not have a zero at $x = 2$. $d(x)$ has a zero at both $x = 2$ and $x = 5$. The multiplicity of the zero at $x = 5$ is greater for $n(x)$ than it is for $d(x)$.

3. $n(x)$ has a zero at $x = 5$ and does not have a zero at $x = 2$. $d(x)$ has a zero at both $x = 2$ and $x = 5$. The multiplicity of the zero at $x = 5$ is greater for $d(x)$ than it is for $n(x)$.

4. $n(x)$ has a zero at $x = 2$ and $x = 5$. $d(x)$ has a zero at $x = 2$ and $x = 5$. The multiplicity of the zero at $x = 2$ is equal for $n(x)$ and $d(x)$. The multiplicity of the zero at $x = 5$ is greater for $n(x)$ than it is for $d(x)$.

A) The graph of f has holes at both $x = 2$ and $x = 5$

B) The graph of f has a vertical asymptote at $x = 2$ and a hole at $x = 5$

C) The graph of f has a hole at $x = 2$ and a vertical asymptote at $x = 5$

D) The graph of f has vertical asymptotes at both $x = 2$ and $x = 5$

For each of the following functions, identify the domain, the equation of all vertical asymptotes, and the coordinates of all holes.

5. $y = \frac{x^2-16}{2x-8}$

6. $y = \frac{x^2+3x-18}{x^2+9x+18}$

7. $y = \frac{x^2+3x-4}{x^2-9x+20}$

8. $y = \frac{x^3+6x^2-27x}{2x^2+7x-15}$

9. $y = \frac{2x^2-5x-3}{x^3+x^2-9x-9}$

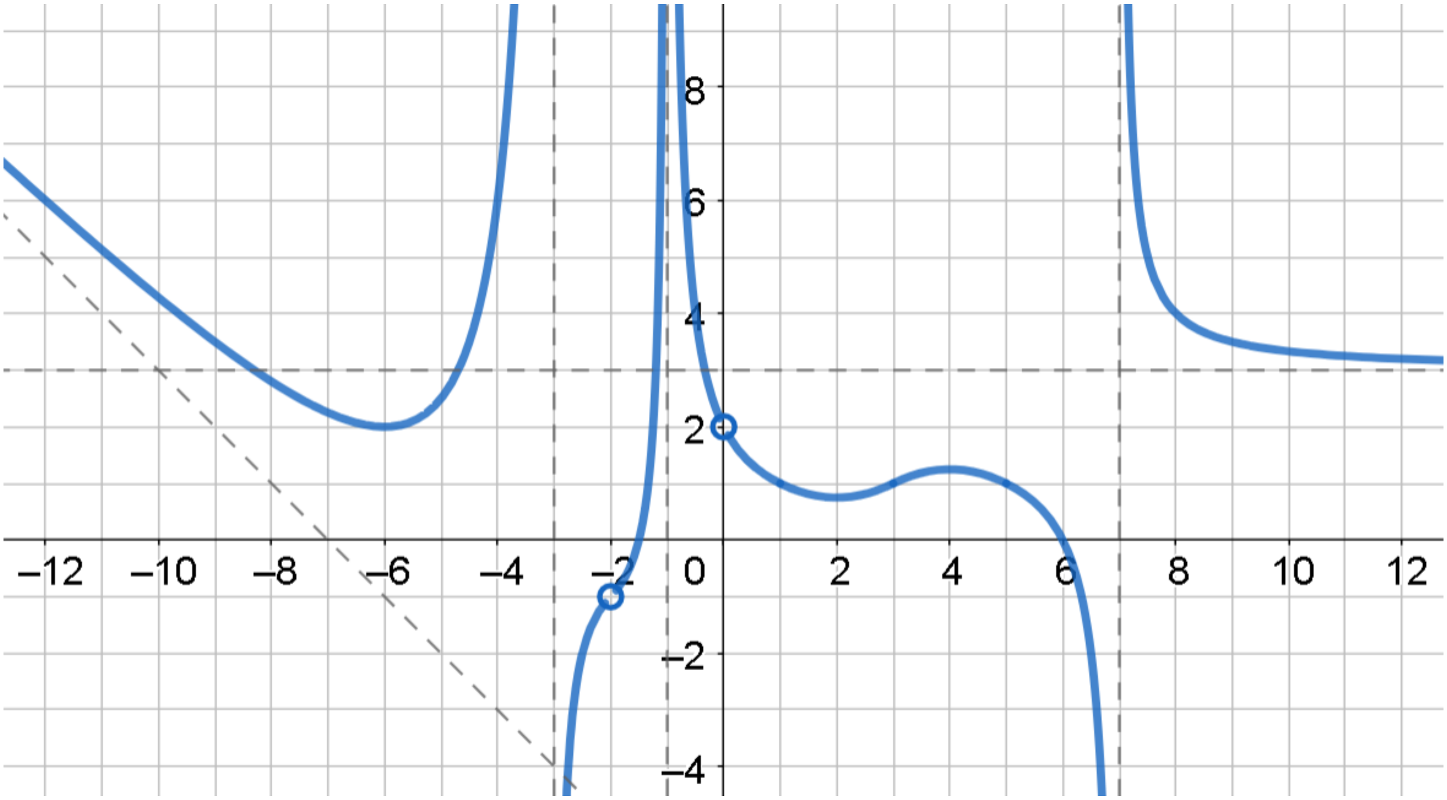
10. $y = \frac{(x+2)}{(x-3)(x+5)^3}$

11. $y = \frac{(x+2)(x+5)}{(x-3)(x+5)^2}$

12. $y = \frac{(x+2)(x+5)^2}{(x-3)(x+5)}$

13. $y = \frac{(x+2)(x+5)^3}{(x-3)}$

Consider $y = f(x)$ graphed below. The function has vertical asymptotes $x = -3$, $x = -1$, and $x = 7$. There is a hole at $(-2, -1)$ and $(0, 2)$. The horizontal asymptote $y = 3$ is approached at the right end of the graph. The left end behavior is unbounded.



14. Find $\lim_{x \to -3^-} f(x)$

15. Find $\lim_{x \to 0^+} f(x)$

16. Find $\lim_{x \to \infty} f(x)$

17. Find $\lim_{x \to -1^-} f(x)$

18. Find $\lim_{x \to 7^-} f(x)$

19. Find $\lim_{x \to -3^+} f(x)$

20. Find $\lim_{x \to -1^+} f(x)$

21. Find $\lim_{x \to 7^+} f(x)$

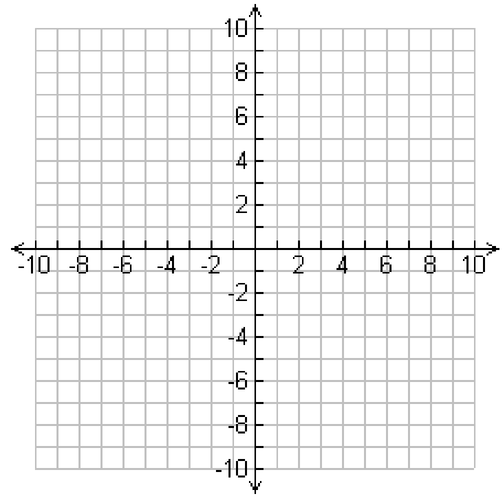
22. Find $\lim_{x \to -2} f(x)$

23. Find $\lim_{x \to 0^-} f(x)$

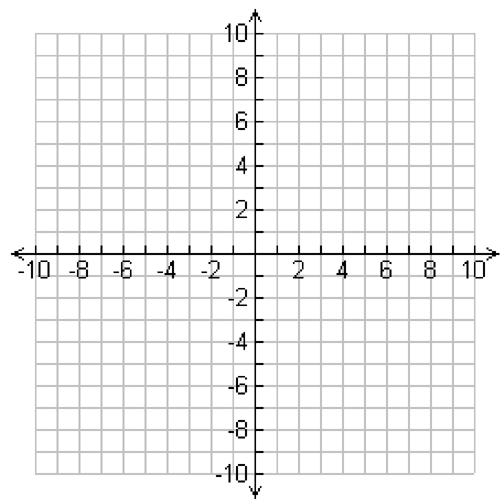
24. Find $\lim_{x \to -\infty} f(x)$

25. While this graph is great for practicing limits, this function cannot possibly be a rational function. Give a reason why not

26. On the graph provided to the right, sketch a rational function $f(x)$ for which the following are true:
   - $\lim_{x\to -1^-} f(x) = \infty$
   - $\lim_{x\to -1^+} f(x) = -\infty$
   - $\lim_{x\to 2^-} f(x) = \infty$
   - $\lim_{x\to 2^+} f(x) = -\infty$
   - $\lim_{x\to -\infty} f(x) = 0$
   - $\lim_{x\to \infty} f(x) = 0$
   - $\lim_{x\to 4} f(x) = -1$



27. On the graph provided to the right, sketch a rational function $f(x)$ for which the following are true:
   - $\lim_{x\to -\infty} f(x) = \infty$
   - $\lim_{x\to 0^-} f(x) = 3$
   - $\lim_{x\to 0^+} f(x) = 3$
   - $\lim_{x\to 1^-} f(x) = \infty$
   - $\lim_{x\to 1^+} f(x) = \infty$
   - $\lim_{x\to \infty} f(x) = -\infty$

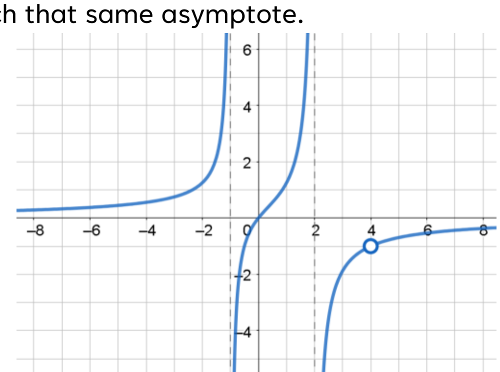


28. Given $f(x) = \frac{x^2+2x-15}{x^2-5x+6}$, find $\lim_{x\to 3} f(x)$.

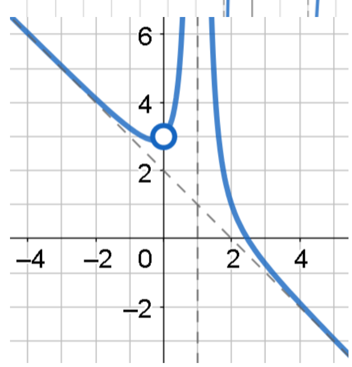
29. Given $g(x) = \frac{2x^2-9x-5}{2x^2+3x+1}$, find $\lim_{x\to -\frac{1}{2}} g(x)$.

Rational Functions and One-Sided Limits (1.9-1.11) HW Answers

1. C
2. B
3. D
4. A
5. Domain: $(-\infty,4)\cup(4,\infty)$ Vertical Asymptotes: None Hole: $(4,4)$
6. Domain: $(-\infty,-6)\cup(-6,-3)\cup(-3,\infty)$ Vertical Asymptotes: $x=-3$ Hole: $(-6,3)$
7. Domain: $(-\infty,4)\cup(4,5)\cup(5,\infty)$ Vertical Asymptotes: $x=4$, $x=5$ Hole: None
8. Domain: $(-\infty,-5)\cup\left(-5,\frac{3}{2}\right)\cup\left(\frac{3}{2},\infty\right)$ Vertical Asymptotes: $x=-5, x=\frac{3}{2}$ Hole: None
9. Domain: $(-\infty,-3)\cup(-3,-1)\cup(-1,3)\cup(3,\infty)$ Vertical Asymptotes: $x=-3, x=-1$ Hole: $\left(3,\frac{7}{24}\right)$
10. Domain: $(-\infty,-5)\cup(-5,3)\cup(3,\infty)$ Vertical Asymptotes: $x=-5, x=3$ Hole: None
11. Domain: $(-\infty,-5)\cup(-5,3)\cup(3,\infty)$ Vertical Asymptotes: $x=-5, x=3$ Hole: None
12. Domain: $(-\infty,-5)\cup(-5,3)\cup(3,\infty)$ Vertical Asymptotes: $x=3$ Hole: $(-5,0)$
13. Domain: $(-\infty,3)\cup(3,\infty)$ Vertical Asymptotes: $x=3$ Hole: None
14. ∞
15. 2
16. 3
17. ∞
18. $-\infty$
19. $-\infty$
20. ∞
21. ∞
22. -1
23. 2
24. ∞
25. The given graph approaches a horizontal asymptote on its right end ($\lim_{x\to\infty} f(x)=3$). If this were a rational function, the left end would have to also approach that same asymptote.
26. While there are other possible correct answers, to the right is a straightforward answer:



27. While there are other possible correct answers, to the right is a straightforward answer. The slant asymptote was not necessary as long as the left end behavior is going up and the right end behavior is going down.



28. 8
29. -11